

Charged Cosmic String Nucleation in de Sitter Space

Minos Axenides¹

*The Niels Bohr Institute
University of Copenhagen, 17 Blegdamsvej, 2100 Copenhagen, Denmark*

Arne L. Larsen²

*Nordita
17 Blegdamsvej, 2100 Copenhagen Ø, Denmark*

PACS numbers: 0420, 1110, 1117

¹e-mail: axenides@nbivax.nbi.dk

²e-mail: allarsen@nbivax.nbi.dk

Abstract

We investigate the quantum nucleation of pairs of charged circular cosmic strings in de Sitter space. By including self-gravity we obtain the classical potential energy barrier and compute the quantum mechanical tunneling probability in the semiclassical approximation. We also discuss the classical evolution of charged circular strings after their nucleation.

1 Introduction

Inflation [1, 2, 3] is a short period of rapid expansion in the early history of the universe, whereby its presently observable part originated from a tiny initial region. Topological defects such as cosmic strings, monopoles and domain walls are extended objects present in the spectrum of grand unified theories, that are believed to be typically generated in phase transitions in the early universe. They could have acted as seeds for the generation of density perturbations that resulted in the large scale structure of the universe and the observed anisotropy in the cosmic microwave background radiation. Recently it was realized that such objects can be created spontaneously in a de Sitter spacetime through the process of quantum nucleation [4]. More specifically, using the static parametrization it was found that the classical evolution of a circular string is determined by a simple potential barrier and that strings can nucleate by a quantum mechanical tunneling through the barrier. In a realistic situation any topological defects formed at the onset of the inflationary period are expected to be inflated away. Strings nucleated towards the inflationary exit will eventually contract upon their entrance into the radiation dominated phase inside the causal horizon [5]. If they are still circular they are expected to form black holes [6], otherwise they oscillate radiating away their gravitational energy.

Charged and/or superconducting strings [7] have also been the subject of intense investigation. This is due to their potential role as seeds of the large scale structure of the Universe [8]. Their dynamical properties have already been analyzed in some detail (see [9-15] and references therein). Their electromagnetic structure is typically represented by a physical degree of freedom in the form of a world-sheet real scalar field Φ . In a more recent analysis of their evolution in de Sitter spacetime, the equations of motion of their circular loop radius was obtained in the form of an "effective" repulsive classical potential [16]. A quantum mechanical process, where a string of radius r_1 tunnels through a local barrier on the overall repulsive potential and becomes a string of radius $r_2 \neq r_1$, was then considered and the tunneling probability was evaluated.

In the present paper we push this picture a little further, in discussing the question of quantum *nucleation* of (pairs of) charged strings in de Sitter space. We carefully consider the interplay of the self-gravitational attraction

and coulombic repulsion of small circular charged string loops. We will argue for the presence of an overall classical potential energy barrier in the radial equation of motion of the string loop in de Sitter spacetime. In a first approximation we subsequently compute the probability for quantum creation of charged strings and analyze their classical evolution.

The paper is organized as follows: in Section 2 we derive the charged circular string evolution equations in de Sitter spacetime. In Section 3 we include the string self-gravity to a first approximation and calculate the nucleation probability for charged strings. Finally we consider briefly the classical evolution of charged strings after nucleation.

2 String Evolution Equations

In this section we present the charged string model and derive the equations of motion for a uniformly charged circular string in the background of de Sitter space.

We will consider strings which in the general case are described by the action:

$$S = - \int d\tau d\sigma \sqrt{-\det G} \left[\mu + \frac{1}{2} G^{\alpha\beta} (\Phi_{,\alpha} + A_\mu X_{,\alpha}^\mu) (\Phi_{,\beta} + A_\mu X_{,\beta}^\mu) \right], \quad (2.1)$$

where μ is the string tension, $G_{\alpha\beta}$ is the induced metric on the world-sheet, Φ is a scalar field on the world-sheet and A_μ is the external electromagnetic potential. The action (2.1) is well studied in the literature [9-15]. It is however usually expressed in the "adjoint" form:

$$S^\dagger = - \int d\tau d\sigma \left[\mu \sqrt{-\det G} + \frac{1}{2} \sqrt{-\det G} G^{\alpha\beta} \Psi_{,\alpha} \Psi_{,\beta} + \epsilon^{\alpha\beta} A_\mu X_{,\alpha}^\mu \Psi_{,\beta} \right], \quad (2.2)$$

where Φ and Ψ are related by:

$$\epsilon^{\alpha\beta} \Psi_{,\beta} = \sqrt{-\det G} G^{\alpha\beta} (\Phi_{,\beta} + A_\mu X_{,\beta}^\mu). \quad (2.3)$$

The above model (in the form of eq.(2.1) or (2.2)) should be considered as a covariant version of the charged and current carrying superconducting cosmic string discovered by Witten in a $U(1) \times U(\tilde{1})$ gauge theory [7].

From the action (2.1) we obtain the following expression for the spacetime electromagnetic current:

$$J^\mu = -\sqrt{-\det G} G^{\alpha\beta} (\Phi_{,\alpha} + A_\nu X_{,\alpha}^\nu) X_{,\beta}^\mu \equiv j \frac{\partial X^\mu}{\partial \sigma} - \rho \frac{\partial X^\mu}{\partial \tau}, \quad (2.4)$$

where j is given by:

$$j = -\sqrt{-\det G} [G^{\sigma\tau} (\dot{\Phi} + A_\nu \dot{X}^\nu) + G^{\sigma\sigma} (\Phi' + A_\nu X'^\nu)]. \quad (2.5)$$

and ρ by:

$$\rho = \sqrt{-\det G} [G^{\tau\tau} (\dot{\Phi} + A_\nu \dot{X}^\nu) + G^{\tau\sigma} (\Phi' + A_\nu X'^\nu)] \quad (2.6)$$

They represent the current and charge density on the world-sheet, respectively. The continuity equation is obtained from the equation of motion for Φ :

$$\dot{\rho} = j'. \quad (2.7)$$

As a special case we now consider the de Sitter background:

$$ds^2 = -(1 - H^2 r^2) dt^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (2.8)$$

supplemented by:

$$A \equiv A_\mu dx^\mu = 0. \quad (2.9)$$

Furthermore, we are only interested in circular strings as obtained by the ansatz:

$$t = \tau, \quad r = r(\tau), \quad \theta = \frac{\pi}{2}, \quad \phi = \sigma, \quad \text{as well as:} \quad \Phi = \Phi(\tau). \quad (2.10)$$

It is straightforward to show that the string equations of motion take the form for Φ :

$$\dot{\Phi} = -\frac{\Omega}{E} (1 - H^2 r^2) \left(\mu + \frac{\Omega^2}{2r^2} \right) \quad (2.11)$$

and for r :

$$r^2 - (1 - H^2 r^2)^2 + \frac{r^2}{E^2} (1 - H^2 r^2)^3 \left(\mu + \frac{\Omega^2}{2r^2} \right)^2 = 0, \quad (2.12)$$

where E and Ω are integration constants. From eqs.(2.5) and (2.6) it follows that:

$$\rho = \Omega, \quad j = 0, \quad (2.13)$$

so that the ansatz (2.10) describes a uniformly charged circular string with charge density Ω and no current. The integration constant E is interpreted as a constant energy density:

$$E = -\frac{1}{2\pi} \int_0^{2\pi} \frac{\delta \mathcal{L}}{\delta \dot{t}} d\sigma, \quad (2.14)$$

which, however, should not be confused with the energy density as obtained from the spacetime energy-momentum tensor $T^{\mu\nu}$. The remaining physical degree of freedom of the string is the radius of the loop which is determined by eq.(2.12). For $\Omega = 0$ it reduces to:

$$\dot{r}^2 - (1 - H^2 r^2)^2 + \frac{r^2}{\epsilon^2} (1 - H^2 r^2)^3 = 0, \quad (2.15)$$

where $\epsilon \equiv E/\mu$, in agreement with the result of Basu, Guth and Vilenkin [4]. In what follows, however, we will only be interested in the case of charged strings ($\Omega \neq 0$). Notice that both eqs.(2.15) and (2.12) can be solved explicitly in terms of elliptic integrals, but the general solutions will not be important here. It is convenient to write eq.(2.12) in the form:

$$\dot{r}^2 + V(r) = 0, \quad (2.16)$$

where:

$$V(r) = (1 - H^2 r^2)^2 \left[\frac{1}{E^2} \left(\mu r + \frac{\Omega^2}{2r} \right)^2 (1 - H^2 r^2) - 1 \right], \quad (2.17)$$

so that the classical motion takes place at the r -axis in a $(r, V(r))$ diagram. The allowed loop trajectories can therefore be extracted from the knowledge about the zeros of the potential $V(r)$. In general we have that $V(H^{-1}) = 0$ and $V(r \rightarrow 0) = \infty$ (for $\Omega \neq 0$), and the equation $V(r) = 0$ reduces to a cubic equation in the variable r^2 . This allows for a complete and explicit classification of the classical motion for any values of the parameters (H, μ, Ω, E) . For our purposes, (see Section 3) it is sufficient to consider the limiting case $E \rightarrow 0$, where the potential takes the form of Fig.1, i.e. a classical circular string with $E = 0$ cannot exist inside the horizon, $r_{\text{hor}} = H^{-1}$.

3 Self-Gravity and Nucleation Probabilities

In the preceding section we derived the potential that determines the loop radius of a charged string in the background of de Sitter space. Independently of the detailed form, the potential always blows up for $r \rightarrow 0$. However, the physical picture discussed in Section 2 cannot be trusted for $r \rightarrow 0$. Besides the assumption that the thickness of the string is infinitesimal, we also have neglected the self-gravity of the string. For a small string-loop the gravitational effects cannot be neglected. More specifically, if the circular string falls into its own Schwarzschild radius, it will inevitably collapse into a black hole. The precise dynamics of a circular string including gravitational and electromagnetic effects (self-gravity, radiation back-reaction, energy loss due to the radiation etc.) is not known. However, we will now argue that the effect of the self-gravity is a "prefactor" in the form of the potential near the value of the Schwarzschild radius of the string. Our argument goes as follows: the dynamics of a circular string with self-gravity in a de Sitter background is somewhat similar (can be approximated) to the dynamics of a circular string without self-gravity in a Schwarzschild de Sitter background with an appropriate value for the mass parameter M (here considered as a free parameter). The metric of such a background is given by:

$$ds^2 = -(1 - \frac{2M}{r} - H^2 r^2) dt^2 + \frac{dr^2}{1 - \frac{2M}{r} - H^2 r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2. \quad (3.1)$$

For $0 \leq HM \leq 1/\sqrt{27}$, the Schwarzschild de Sitter space has a de Sitter event horizon (r_+) and a Schwarzschild horizon (r_-) which are given by:

$$Hr_+ = \frac{1}{3^{\frac{1}{3}} Z(HM)} + \frac{Z(HM)}{3^{\frac{2}{3}}} \quad (3.2)$$

and:

$$Hr_- = \frac{1}{2}(i\sqrt{3} - 1) \frac{1}{3^{\frac{1}{3}} Z(HM)} - \frac{1}{2}(i\sqrt{3} + 1) \frac{Z(HM)}{3^{\frac{2}{3}}}, \quad (3.3)$$

where:

$$Z(HM) \equiv [-9HM + \sqrt{3(-1 + 27H^2 M^2)}]^{\frac{1}{3}}. \quad (3.4)$$

The range of values $HM > 1/\sqrt{27}$ is unphysical as it corresponds to a space-time with a naked singularity. In what follows we will restrict ourselves to

the values $0 < HM < 1/\sqrt{27}$. By a direct generalization of our treatment of charged circular strings in de Sitter space in the previous section, the equation of motion for the loop radius can be put in the desired form $\dot{r}^2 + V(r) = 0$, where the potential $V(r)$ is now given by:

$$V(r) = (1 - H^2 r^2 - \frac{2M}{r})^2 [\frac{1}{E^2}(\mu r + \frac{\Omega^2}{2r})^2 (1 - H^2 r^2 - \frac{2M}{r}) - 1]. \quad (3.5)$$

It follows that $V(r_{\pm}) = 0$, i.e. the potential from strongly repulsive becomes locally attractive for $r \rightarrow 0$. For $M \ll H^{-1}$ we find the approximate expressions

$$Hr_+ = 1 - HM + \mathcal{O}(H^2 M^2) \quad (3.6)$$

and:

$$Hr_- = 2HM + \mathcal{O}(H^2 M^2), \quad (3.7)$$

and the potential (3.5) takes the form of Fig.2 in the limit $E \rightarrow 0$. In effect, self-gravity creates a finite potential energy barrier for arbitrary values of the conserved energy E . A charged circular string has thus the possibility to tunnel quantum mechanically through the barrier. More importantly in the limit of $E \rightarrow 0$ such a tunneling process can be interpreted as a spontaneous nucleation of charged circular strings. Because of charge conservation such charged strings must be created in pairs spontaneously, and then evolve by flying apart from each other. The potential barrier derived in this section is responsible for the quantum creation process; a tunneling from nothing into a finite radius circular loop for each of the string loops. It must be stressed that this nucleation process can only be modelled through the inclusion of self-gravity of each of the circular charged strings. This is evident by a direct comparison of Figs.1 and 2. The probability for nucleation can now be evaluated in the WKB approximation:

$$\mathcal{T} \propto e^{-B}, \quad (3.8)$$

where:

$$B = 2 \int_{r_1}^{r_2} |P_r| dr, \quad (3.9)$$

with:

$$P_r = \int_0^{2\pi} \frac{\delta \mathcal{L}}{\delta \dot{r}} d\sigma. \quad (3.10)$$

Here r_1 and r_2 are the turning points and \mathcal{L} is the Lagrangian that describes the charged string. As we have already pointed out, the exact dynamics of

the circular strings is not known. The Schwarzschild de Sitter model that led to the potential (3.5) is therefore to be taken as an approximation only. A similar approximation, which we will use in what follows, is based on the de Sitter model that naturally led to the potential (2.17), but with a physical cut-off (due to the self-gravity) that pulls down the rising repulsive potential at some value of the radius $r = \delta \ll H^{-1}$. In the following we consider δ as a parameter to be measured experimentally. In eqs.(3.8) – (3.10) we then get:

$$r_1 = \delta, \quad r_2 = H^{-1}, \quad |P_r| = 2\pi \frac{\mu r + \Omega^2/2r}{\sqrt{1 - H^2 r^2}}, \quad (3.11)$$

in the limit $E \rightarrow 0$. It follows that :

$$B = \frac{4\pi\mu}{H^2} \sqrt{1 - H^2 \delta^2} + 2\pi\Omega^2 \ln \frac{1 + \sqrt{1 - H^2 \delta^2}}{H\delta}. \quad (3.12)$$

For $H\delta \ll 1$ we find that:

$$B = -2\pi\Omega^2 \ln(H\delta) + 2\pi \left(\frac{2\mu}{H^2} + \Omega^2 \ln 2 \right) + \mathcal{O}(H^2 \delta^2), \quad (3.13)$$

i.e. :

$$\mathcal{T} \approx (H\delta)^{2\pi\Omega^2} \exp \left[-\frac{4\pi\mu}{H^2} \left(1 + \frac{\Omega^2 H^2}{2\mu} \ln 2 \right) \right]. \quad (3.14)$$

For $\delta = \Omega = 0$, we observe that the nucleation amplitude reduces to the one for the uncharged string [4] (as it should). In the more general case under consideration here it is convenient to introduce the dimensionless variables:

$$x \equiv H\delta \ll 1, \quad y \equiv H^2 \Omega^2 / \mu \geq 0 \quad (3.15)$$

and to plot the function:

$$F(x, y) = \frac{H^2}{2\pi\mu} \ln \frac{\mathcal{T}_{BGV}}{\mathcal{T}} = 2(-1 + \sqrt{1 - x^2}) + y \ln \frac{1 + \sqrt{1 - x^2}}{x}, \quad (3.16)$$

where $\mathcal{T}_{BGV} = e^{-4\pi\mu/H^2}$ (see Fig.3.)

Some comments are now in order:

1. The original Basu-Guth-Vilenkin (BGV) result [4] corresponds to the point $x = y = 0$.
2. For a zero charge string ($y = 0$) the nucleation probability is not sensitive to the physical cut-off δ , i.e. the BGV result is essentially independent of the self-gravity of the string.
3. For a charged string ($y \neq 0$) the nucleation probability is extremely sensitive to the cut-off. More specifically it goes to zero as a power of x for $x \rightarrow 0$.
4. By now combining 2 and 3, $F(x, y)$ is singular along the y -axis, where $\mathcal{T} = 0$, except for $y = 0$ where $\mathcal{T} = \mathcal{T}_{BGV}$. On the other hand, $F(x, y)$ is regular on the x -axis for $0 \leq x < 1$ (where x is defined).

With the above comments our discussion of the charged string nucleation probability is complete. We will finish by making a few observations about the classical evolution of circular charged strings after nucleation. As they nucleate at horizon size, their evolution is most conveniently described in the comoving time :

$$T = \frac{\ln |1 - H^2 r^2|}{2H} + t. \quad (3.17)$$

Notice that r equals the physical radius as seen in spatially flat Robertson-Walker coordinates. After a little algebra we find from eq.(2.12) when $E \rightarrow 0$:

$$\frac{dr}{dT} = -\frac{1 - H^2 r^2}{Hr}, \quad (3.18)$$

which is solved by:

$$H r(T) = \sqrt{1 + e^{2H(T-T_0)}}, \quad (3.19)$$

thus the string expands with the same rate as that of the universe (for $H(T - T_0) \gg 1$). Interestingly, this result is independent of both the string charge Ω and the string tension μ . In fact, the physics here is identical with the evolution of the uncharged circular string as originally was discussed by Basu, Guth and Vilenkin [4]. The reason is that the different parts of the string loop once outside the horizon are not causally connected. The system simply follows the expansion of the universe.

4 Conclusion

In the present work we investigated the dynamics of circular charged strings in a de Sitter spacetime background. We derived the radial equations of motion and identified the repulsive classical potential energy due to the rapid expansion of the background. We included the effect of self-gravity for small radius of the loop by deriving the corresponding equations of motion in a Schwarzschild de Sitter background. We demonstrated the appearance of a classical potential energy barrier and computed the probability for a quantum tunneling of a single uniformly charged loop with zero energy.

As explained in Section 3, we interpret such a spontaneous nucleation effect as the quantum creation of a pair of oppositely charged circular string loops at the cosmic horizon and their subsequent parting off.

5 Acknowledgements

The research of M.A. was partially supported by Danmarks Grund-Forsknings Fond through its support for the establishment of the Theoretical Astrophysics Center.

References

- [1] A. Linde, ‘Particle Physics and Inflationary Cosmology’ (Harwood, Chur, 1990).
- [2] E.W. Kolb and M.S. Turner, ‘The Early Universe’ (Addison-Wesley 1989).
- [3] R.H. Brandenberger in ‘Lectures in Modern Cosmology and Structure Formation’, proc. of the 7th Swieca Summer School (World Scientific, Singapore, 1993).
- [4] R. Basu, A.H. Guth and A. Vilenkin, Phys.Rev.D44 (1991) 340.
- [5] J. Garriga and A. Vilenkin, Phys.Rev.D47 (1993) 3265.
- [6] S.W. Hawking, Phys.Lett.B246 (1990) 36.
- [7] E. Witten, Nucl.Phys.B291 (1985) 557.
- [8] J.P. Ostriker, C. Thomson and E. Witten, Phys.Lett.B180 (1986) 231.
- [9] B. Carter, Phys.Lett.B224 (1989) 61.
- [10] E. Copeland, M. Hindmarsh and N. Turok, Phys.Rev.Lett.58 (1987) 1910.
- [11] A.P. Balachandran, B.S. Skagerstam and A. Stern, Phys.Rev.D20 (1979) 439.
- [12] A. Davidson and K.C. Wali, Nucl.Phys.B348 (1991) 581.
- [13] A. Vilenkin and T. Vachaspati, Phys.Rev.Lett.58 (1987) 1041.
- [14] D.N. Spergel, T. Piran and J. Goodman, Nucl.Phys.B291 (1987) 847.
- [15] A.L. Larsen, Class.Quant.Grav.10 (1993) 1541.
- [16] A. Davidson, N.K. Nielsen and Y. Verbin, Nucl.Phys.B412 (1994) 391.

Figure Captions

Fig.1. The potential (2.17) determining the loop-radius of a charged string in de Sitter space. Shown is the case where $E = 0$. The string motion is confined to the r -axis, i.e. the string cannot exist inside the horizon.

Fig.2. The potential (3.5) determining the loop-radius of a charged string in Schwarzschild de Sitter space. As in Fig.1 we consider only the case where $E = 0$.

Fig.3. The function $F(x, y)$ (eq.(3.16)) determining the nucleation probability of charged strings in de Sitter space when including the self-gravity. Notice that $F = 0$ corresponds to $\mathcal{T} = \mathcal{T}_{BGV}$ and $F = \infty$ corresponds to $\mathcal{T} = 0$.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-th/9409005v1>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-th/9409005v1>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/hep-th/9409005v1>